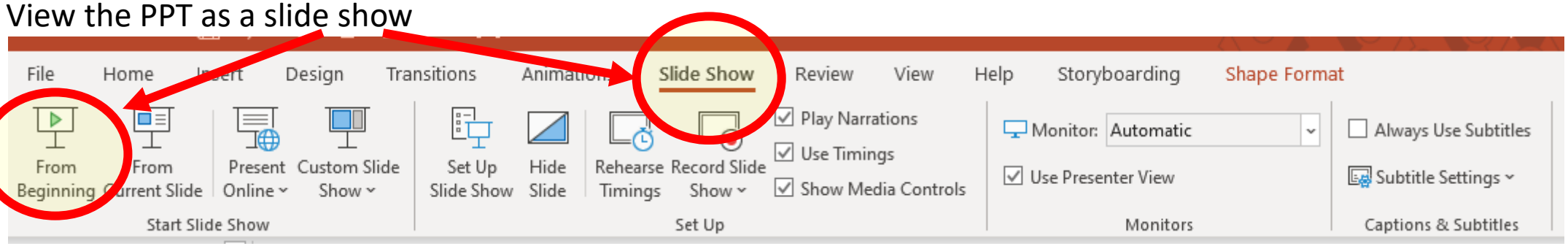
How to best use these slides…

- View the PPT as a slide show



- Then click through every step
  - Mouse clicks will advance the slide show
  - Left/right arrow keys move forward/backward
  - Mouse wheel scrolling moves forward/backward
- When a question is posed, stop and think it through, try to answer it yourself before clicking
- If you have questions, use PS discussion boards, email me, and/or visit us in a Teams class session!

LESSON 7.1a

Direct and Inverse Variation

Today you will:

- Review direct variation and the constant of variation
- Learn what inverse variation is
- Classify equations and data as direct or inverse variation (or neither!)
- Practice using English to describe math processes and equations

Core Vocabulary:

- Inverse variation, p. 360
- Constant of variation, p 360

Previous:

- Direct variation
- Ratios

What do you think?

Soccer: the harder I kick the ball, the <u>further</u> the ball goes. **first ↑ second ↑**

Weightlifting: the more weight I put on the bench press bar, the <u>fewer</u> times I can press it. **first ↑ second ↓**

Driving: the harder I press the gas peddle (accelerator), the <u>faster</u> the car goes. **first ↑ second ↑**

Driving: the harder I press the gas petal, my gas mileage <u>goes down</u>. **first ↑ second ↓**

These are all examples of what we call variation or variance.

- Two things that are related … one affects the other
- In some cases, as the first increases ↑, the second also increases ↑ ***direct variation***
- In other cases, as the first increases ↑, the second decreases ↓ ***inverse variation***

What *direct* and *inverse* variation look like in algebra

Relationship between two "things" that can change/vary
…let's call the 1st x
…and let's call the 2nd y

Direct Variation	**Inverse Variation**
• Also called ***directly proportional***	• As the 1st increases the 2nd decreases
• As the 1st increases, the 2nd increases	• As the 1st decreases, the 2nd increases
• As the 1st decreases, the 2nd decreases	• They "scale inversely" … opposite
• They "scale directly"	• $y = \frac{a}{x}$
• $y = ax$	

What's up with the a? (most of the math world uses k)

- It is called the ***constant of variation***
- It is basically the "connection" between the two variables
- It is the scaling factor
- Shows how much bigger or smaller one is than the other

How do you tell if an equation is direct or inverse variation (or neither)?

1. Get x and y on opposite sides of the equation (solve for y)
2. Check if they are both on the top (direct variation)

   …or one on the top and the other on the bottom (inverse variation)
3. If there is something added or subtracted from x, it is neither

Direct Variation	Inverse Variation	Neither
$y = ax$ Both variables "on the top" Examples: ▸ $y = x$ ▸ $y = 3x$ ▸ $y = \frac{x}{2}$ ▸ $\frac{y}{x} = 4$	$y = \frac{a}{x}$ One variable on the top (y) …the other (x) on the bottom Examples: ▸ $y = \frac{1}{x}$ ▸ $y = \frac{4}{3x}$ ▸ $x = \frac{2}{y}$ ▸ $xy = 5$	Examples: ▸ $y = x + 3$ ▸ $y = \frac{2}{x} - 5$

Tell whether x and y show *direct variation, inverse variation,* or *neither*.

a. $xy = 5$

b. $y = x - 4$

c. $\frac{y}{2} = x$

SOLUTION

STUDY TIP

The equation in part (b) does not show direct variation because $y = x - 4$ is not of the form $y = ax$.

Given Equation	Solved for y	Type of Variation
a. $xy = 5$	$y = \frac{5}{x}$	inverse
b. $y = x - 4$	$y = x - 4$	neither
c. $\frac{y}{2} = x$	$y = 2x$	direct

How do you tell by looking at the data if they represent direct or inverse variation (or neither)?

Here are the general equations for both types of variation:

Direct: $y = ax$

Inverse: $y = \frac{a}{x}$

Do you remember what we call a? The constant of variation.

What does it tell us? How the two variables are related.

When you look at data, you will often be given the data in a table like this:

x	1	2	3	4
y	3	6	9	12

Can you think of a way we could use the data provided to tell if this is direct variation, inverse variation or neither?

Hints:

- Ask yourself what information is provided? $\boldsymbol{x \text{ and } y}$
- Ask yourself what information is missing? $\boldsymbol{a}$ **(the constant of variation)**
- The BIG question: **Can $\boldsymbol{a}$ help us determine which variation type the data represents?**

How do you tell by looking at the data if they represent direct or inverse variation (or neither)?

Can a help us classify this data? **YES**

Here is the data again:

x	1	2	3	4
y	3	6	9	12

…and here are the general equations for both types of variation:

Direct: $y = ax$

Inverse: $y = \frac{a}{x}$

Since a is the key, let's rewrite the two general equations in terms of a … i.e. solve for a

Direct: $a = \frac{y}{x}$

Inverse: $a = xy$

What does this tell us? How does this help us?

Given x & y data:

- If the ***ratio*** of x & y is a constant (every x, y pair ***divided*** gives us the same number)
  - then the data directly varies
- If the ***product*** of x & y is a constant (every x, y pair ***multiplied*** gives us the same number)
  - then the data inversely varies

Tell whether x and y show *direct variation, inverse variation,* or *neither*.

a.

x	2	4	6	8
y	−12	−6	−4	−3

b.

x	1	2	3	4
y	2	4	8	16

SOLUTION

a. Find the products xy and ratios $\frac{y}{x}$.

products	xy	−24	−24	−24	−24
ratios	$\frac{y}{x}$	$\frac{-12}{2} = -6$	$\frac{-6}{4} = -\frac{3}{2}$	$\frac{-4}{6} = -\frac{2}{3}$	$-\frac{3}{8}$

The products are constant.

The ratios are not constant.

▶ So, x and y show inverse variation.

ANALYZING RELATIONSHIPS

In Example 2(b), notice in the original table that as x increases by 1, y is multiplied by 2. So, the data in the table represent an exponential function.

b. Find the products xy and ratios $\frac{y}{x}$.

products	xy	2	8	24	64
ratios	$\frac{y}{x}$	$\frac{2}{1} = 2$	$\frac{4}{2} = 2$	$\frac{8}{3}$	$\frac{16}{4} = 4$

The products are not constant.

The ratios are not constant.

▶ So, x and y show neither direct nor inverse variation.

Review/recap

Direct variation:
- $y = ax$
- Variables both "on top"
- Both variables increase/decrease in the same "direction"

Inverse variation:
- $y = \frac{a}{x}$
- One variable on the top, the other on the bottom
- The variables increase/decrease in opposite directions

Constant of variation:
- a
- Tells us how the two variables relate
- Is the "scaling factor"

Don't get confused!!!
- If we're looking at ***equations***: check to see if the variables are both on the top, or if one is on top, the other on the bottom
- If we're looking at ***data***: check to see if the product or ratio of the x & y values are constant

These Khan Academy video are great! Make sure you check them out:
- Intro to direct & inverse variation
- Recognizing direct & inverse variation
- Practice: Recognize direct & inverse variation
- Recognizing direct & inverse variation: table (he tosses in "joint variation" … variation between more than two variables)

Homework

Pg 363, #3-14